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through an angle of 180°, and it will become the vector $(+1)$, or will be multiplied by (-1); that is, $(-1)(-1)=+1$. [*F. P. Matz.*]

PROBLEMS.

56. Proposed by CHAS. E. MYERS, Canton, Ohio, and Hon. JOSIAH H. DRUMMOND, LL. D., Portland, Maine.

(a), How much can be paid for a bond, bearing 5% interest and having ten years to run, so as to realize 3% on the investment? [*C. E. Myers*]; (b), At what price must the government sell 5% \$100 bonds to run ten years, interest payable annually, to make them the same to the buyer as 3% bonds at par, to run ten years, interest payable anually, provided the buyer can invest all interest received at 4% interest payable anually? [*J. H. D.*]

57. Proposed by J. C. CORBIN, Pine Bluff, Arkansas.

Find the quotient of

$$\begin{vmatrix} (s-a_1)^2 & a_1^2 & a_1^2 \ldots\ldots\ldots a_1^2 \\ a_2^2 & (s-a_2)^2 & a_2^2 \ldots\ldots\ldots a_2^2 \\ a_3^2 & a_3^2 & (s-a_3)^2 \ldots\ldots\ldots a_3^2 \\ \ldots\ldots & \ldots\ldots & \ldots\ldots\ldots\ldots\ldots\ldots \\ a_n^2 & a_n^2 & a_n^2 \ldots\ldots s-a_n^2 \end{vmatrix} \div \begin{vmatrix} s-a_1 & a_1 & a_1 \ldots\ldots\ldots a_1 \\ a_2 & s-a_2 & a_2 \ldots\ldots\ldots a_2 \\ a_3 & a_3 & s-a_3 \ldots\ldots\ldots a_3 \\ \ldots\ldots & \ldots\ldots & \ldots\ldots\ldots\ldots\ldots\ldots \\ a_n & a_n & a_n \ldots\ldots s-a_n \end{vmatrix}$$

GEOMETRY.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

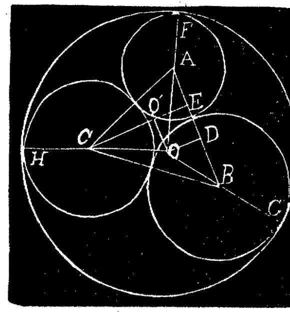
45. Proposed by B. F. BURLESON, Oneida Castle, New York.

Determine the radius of a circle circumscribing three tangent circles of a radii $a=15$, $b=17$, and $c=19$.

I. Solution by the PROPOSER; J. F. W. SCHEFFER, A. M., Hagerstown, Maryland; A. H. BELL, Hillsboro, Illinois; and F. P. MATZ, M. Sc., Ph. D., Mechanicsburg, Pennsylvania.

The problem has two cases: first, when the three given circles are tangent internally to the required circle, as in the problem; and, second, when the required circle is tangent to them externally. But *one solution* involving the resolution of a quadratic equation, will give the answers to both cases. We give the figure for the *first* case only.

Join the centers of the three given circles forming the triangle ABC. Put $AF=a=15$ ft., $CH=b=17$ ft., and $BG=c=19$ ft. Draw CE perpendicular to AB. Let O be the center of the required fourth tangent circle. Draw the radii $R=OCH=OAF=OBG$. Drop on CE the perpendicular OO', and on AB the perpendicular OD. We have $AC=a+b$, $AB=a+c$ and $BC=b+c$. It is evident that $AO=R-a$, $BO=R-c$, and $CO=R-b$. We have in the triangle ABC, by geometry: $AB:BC+AC::BC-AC:BE-AE$; that is, $c+a:a+2b+c::c-a:(a+2b+c)(c-a)=BE-AE$.



$\therefore$ $AE=(a^2+ac+ab-bc)\div(c+a)$, and $BE=(c^2+ac+bc-ab)\div(c+a)$. Again in the triangle AOB, we have $AB:AO+BO::AO-BO:AD-BD$; that is, $c+a:2R-(c+a)::c-a:(2R-a-c)(c-a)\div(c+a)=AD-BD$. $\therefore$ $AD=(a^2+ac+Rc-Ra)\div(c+a)$, and $BD=(c^2+ac-Rc+Ra)\div(c+a)$. Now $OO'=DE=BE-BD=(bc-ab+Rc-Ra)\div(c+a)$; also $EO'=DO=\sqrt{(BO^2-BD^2)}=2\sqrt{(R^2ac-Rac^2-Ra^2c)}\div(c+a)$. The perpendicular $CE=2\sqrt{[abc(a+b+c)]}\div(c+a)$. Now $CO'=CE-EO'=\{2\sqrt{[abc(a+b+c)]}-2\sqrt{(R^2ac-Rac^2-Ra^2c)}\}\div(c+a)$. Again $CO=\sqrt{(CO'^2+OO'^2)}=\sqrt{\{4a^2bc+4ab^2c+4abc^2+4R^2ac-4Rac^2-4Ra^2c+(bc-ab+Rc-Ra)^2-8\sqrt{[abc(a+b+c)(R^2ac-Rac^2-Ra^2c)]}\}}\div(c+a)$. Putting this value of $CO=R-b$ and clearing the resulting equation from radicals by two successive involutions, we obtain the quadratic equation, after dividing by $(c+a)^2$; $[2abc(a+b+c)-(a^2b^2+a^2c^2+b^2c^2)]R^2-2abc(ab+ac+bc)R=a^2b^2c^2$.

Whence by resolution,

$$R=\frac{abc(ab+ac+bc)\pm2abc\sqrt{abc(a+b+c)}}{2abc(a+b+c)-(a^2b^2+a^2c^2+b^2c^2)}=\frac{4181235\pm494190\sqrt{95}}{243611}$$

$=36.93594828+$ feet, or $-2.60880378+$ ft. The negative value of R, regarded as a positive quantity, is the radius of the circle that is tangent to the three given circles *externally*.

II. **Solution by G. B. M. ZERR, A. M., Ph D., Vice President and Professor of Mathematics in the Texarkana College, Texarkana, Arkansas.**

Let $BG=a,=19$, $CH=b=17$, $AF=c=15$, $OG=OH=OF=r$.

$$\therefore \cos BCA=\frac{b^2+ab+bc-ac}{b^2+ab+bc+ac}=\frac{97}{192},$$

$$\cos BCO=\frac{b^2+ab+ar-br}{ar+br-b^2-ab}=\frac{306+r}{18r-306},$$

$$\cos ACO=\frac{b^2+bc+cr-br}{cr+br-b^2-bc}=\frac{272-r}{16r-272}.$$

But $\cos BCA=\cos(BCA+BCO)$.

$$\therefore r=\frac{abc\{ab+ac+bc\pm2\sqrt{abc(a+b+c)}\}}{4abc(a+b+c)-(ab+ac+bc)^2}=\frac{\{863\pm102\sqrt{95}\}4845}{243611};$$

$\therefore r=36.93595$ or -2.608803.

The first value is the radius of circumscribing circle, the second is the radius of the circle inscribed in the space enclosed by the three circles.

PROBLEMS.

50. Proposed by B. F. FINKEL, A. M., Professor of Mathematics and Physics in Drury College, Springfield, Missouri.

Draw a line perpendicular to the base of a triangle dividing the triangle in the ratio of m to n.

51. Proposed by G. B. M. ZERR, A. M., Ph. D., Vice President and Professor of Mathematics in the Texarkana College, Texarkana, Arkansas.

To construct a trapezoid; given the bases, the perpendicular distance between the bases and the angle formed by the diagonals.

CALCULUS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him

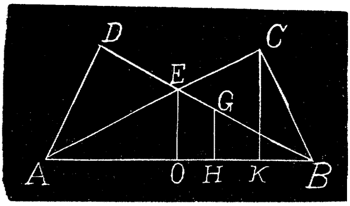
SOLUTIONS OF PROBLEMS.

36. Proposed by H. C. WHITAKER, B. Sc., M. E., Professor of Mathematics, Manual Training School, Philadelphia, Pennsylvania.

A cube is revolved on its diagonal as an axis. Define the figure described and calculate its volume.

Solution by G. B. M. ZERR, A. M., Ph. D., Vice President and Professor of Mathematics in the Texarkana College, Texarkana, Arkansas.

The surface of the solid is that generated by revolving $ADECB$ about AB as an axis.



Let $AD=BC=a$ be the side of the cube. Then $AC=BD=a\sqrt{2}$, $AB=a\sqrt{3}$, $OC=\frac{a\sqrt{3}}{2}$. $AC^2=AB.AK$. $\therefore AK=\frac{2a}{\sqrt{3}}$. $CK^2=AK.BK=AK(AB-AK)$.

$$\therefore CK=\frac{a\sqrt{2}}{\sqrt{3}}.$$

$$GH=\tfrac{1}{3}CK=\frac{a\sqrt{2}}{3\sqrt{3}}.\quad EO:AO=CK:AK.\quad\therefore EO=\frac{a\sqrt{6}}{4}.$$